Sample Mathematics Paper (2 Unit)

(1) Find: $\sqrt[4]{7.4^2}$ correct to 2 decimal places

(2) Factories: $x^2 - 7xy + 12y^2$

(3) (i) Sketch the graph for $y = |3x - 1|$

(4) (ii) Find the *two* values of x such that $|3x - 1| = x$

(5) Find the limiting sum of the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

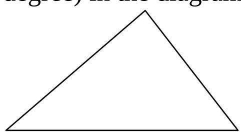
(6) Given that $\log_5 x = k$, express in terms of k, $\log_5 x^2$

(7) Solve the equation $\sqrt{2x - 4} = 6$

(8) Solve $\cos x = -\frac{1}{2}$ for $0 \le x \le 2\pi$

(9) Evaluate $\sum_{r=1}^{8} (5r - 7)$

(10) Find the value of β (nearest degree) in the diagram.



(11) Solve $5 - 3x < 7$

(12) Find the coordinates of the focus of the parabola $12y = x^2 - 4x - 3$.

(13) (i) Show that

$$\frac{d(1 + x^3)^{\frac{1}{2}}}{dx} = \frac{3}{2} \frac{x^2}{\sqrt{1 + x^3}}$$

(ii) Use the result in (i) to find the value of $\int_0^2 \frac{x^2}{\sqrt{1 + x^3}} dx$

(14) (i) Find the equation of the tangent to the curve $y = 3\ln x + 4$ y at the point $x = 1$

(ii) Find the gradient of the normal to the curve at the point $x = 1$

(15) Differentiate with respect to x:

(i) $\ln\left(\frac{5}{x}\right)$ (ii) $x^2 \tan x$ (iii) $3xe^{-2x}$

(iv) $(x^4 - 1)^3$ (v) $\frac{x}{\sin 3x}$ (vi) $x \ln x$

(16) Find:

(i) $\int (2x + 3)^4 dx$ (ii) $\int \left(1 + \frac{3}{4x}\right) dx$

(iii) $\int e^{3x-2} dx$ (iv) $\int_0^{\frac{\pi}{2}} 2\cos\frac{x}{2} dx$

(17) Show that

$$\frac{x^2 + 6x + 7}{(x + 2)(x + 3)} = 1 - \frac{1}{(x + 2)} + \frac{2}{(x + 3)}$$

Hence show that

$$\int_0^2 \left(\frac{x^2 + 6x + 7}{(x + 2)(x + 3)} \right) dx = 2 + \ln\left(\frac{25}{18}\right)$$

(18) Solve:

(i) $2^{2x} - 5(2^x) + 4 = 0$

(ii) $\log_e(x - 2) = 3 - \log_e x$

(19) $f'(x) = 2x^2 + x$. Find $y = f(x)$ if the function passes through (1,5)

(20) The roots of $2x^2 + 6x + 3$ are α and β. Show that $\alpha^2 + \beta^2 = 6$.

(21) For what range of real values k does $kx^2 + kx - 2 = 0$ have the real roots? If the roots are α and β, determine the value of k for which $\alpha = -5/\beta$

(22) For the geometric series $a + ar + ar^2 + \ldots$ the sum of the first two terms is 24 and the sum to infinity is 27.

(i) Show that $r = \pm\frac{1}{3}$

(ii) Find two possible values of *a*.

(23) Solve $4\sin^2 x + 4\cos x - 1 = 0$ for $0^o \le \theta \le 360^o$.

(24) The point A has coordinates (2,-5). The straight line $3x+4y-36=0$ cuts the x-axis at B and the $y-$axis at C. Find:

(i) the equation of the line through A which is perpendicular to the line BC;

(ii) the perpendicular distance from A to the line BC;

(iii) the area of triangle ABC.

(25) Find the values of k for which the equation $2x^2+4x+k=0$ has real roots.

(26) The equation $x^2+6x+1=k\left(x^2+1\right)$ has equal roots. Find the possible values of the constant k.

(27) A curve is given by the equation $y=x^3+bx^2+cx$.

(i) Write down an expression for $\frac{dy}{dx}$. The curve has stationary points when $x=-1$ and $x=3$.

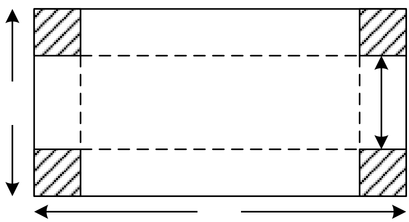
(ii) Show that $b=-3$ and calculate the value of c.

(iii) Hence find the local maximum and minimum values of y.

(28) Prove the following identity:

$$\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=2\sec^2\theta.$$

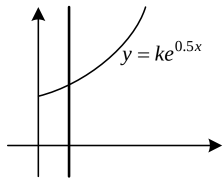
(29) The lengths of a sheet of metal are 8 cm and 3 cm. A square of side x cm is cut from each corner of the sheet and the remaining piece is folded to make an open box.



(i) Show that the volume V of the box is given by $V=4x^3-22x^2+24x$ cm^3.

(ii) Find the value of x for which the volume of the box is a maximum. Calculate the maximum volume.

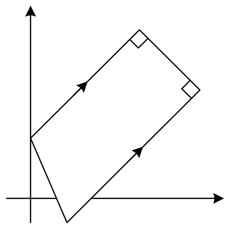
(30) The figure shows $y=ke^{0.5x}$.



(i) Find the value of k.

(ii) The line with equation $x=1$ intersects the graph at P. Find the coordinates of the point P.

(31) The trapezium $ABCD$ has AB parallel to DC. The point A lies on the y-axis. Points B and D are (6,13) and (1,-2) respectively. Angles ABC and BCD are 90°.



Given that the equation of DC is $3y-4x-10=0$, find:

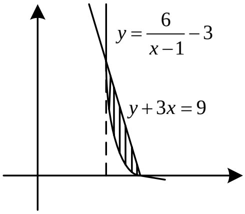
(i) the equations of AB and BC,

(ii) the coordinates of A and C,

(iii) the area of the trapezium.

(32) The line $y=2x$, between the bounds $x=0$ and $x=1$, is rotated about the x axis. Estimate the volume of the solid formed using Simpson's rule with four sub-intervals.

(33) Calculate and show that the shaded area between the curve $y = \frac{6}{x-1} - 3$ and the straight line $y + 3x = 9$ is given by $\frac{3}{2}(3 - 4\ln 2)$



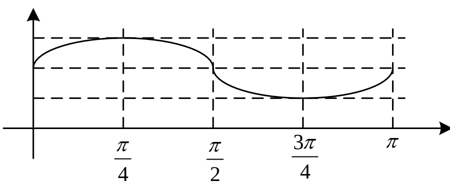
(34) In a medical treatment 500 mg of a drug are administered to a patient. At time t hours, X mg of the drug remains in the patient. The doctor has a mathematical model which states that $X = 500e^{-\frac{1}{5}t}$.

(i) Find the values of t, correct to two decimal places, when $X = 200$.

(ii) Express $\frac{dX}{dt}$ in terms of t.

(iii) Show that when $X = 200$ the rate of decrease of the amount of the drug remaining in the patients is 40 mg per hour.

(35) The diagram shows the graph of the function $y = a + b\sin cx$ for $0 \le x \le \pi$

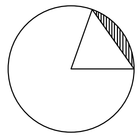


(i) Write down the values of a, b, and c.

(ii) Find algebraically the values of x for which` $y = 2.5$.

(36) A curve has the equation $y = x\sin 2x$. Find the gradient of the curve at $x = \pi/3$.

(37) The diagram shows a circle centre O, radius 10 cm. AB is a chord and angle AOB is θ radian. Use the formula for the area of a sector to show that the area of the minor segment cut off by the cord AB (and shown shaded in the diagram) is $50(\theta - \sin\theta)$ cm^2.



(i) Show that $\sin\theta = \theta - \frac{1}{10}\pi$.

(ii) Check that $\theta = 1.27$ satisfies the equation approximately.

(iii) Deduce the approximate value of angle AOB in degrees.

(38) A particle moves in a straight line so that its distance from a fixed point O after t seconds is s metres where $s = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t$. Show that the particle is at rest when $t = 1$ and $t = 2$ seconds. Find the acceleration of the particle at these times and interpret the results.

(39) Find the coordinates of the stationary points on the curve $y = x^4 - 3x^3$. Show that the curve has a point of inflexion at $x = 2$.

(40) A man invests \$100 at the beginning of each year for ten years. The rate of compound interest is 9% per annum. Calculate the total value of the investment at the end of the ten full years.

(41) Use Simpson's rule with five function values to obtain an approximate value for $\int_0^{90} \sin\frac{\pi x}{180}\,dx$.

Evaluate the integral directly.