Sample Maths Paper (Extension 1) suitable for HSC students

Question 1

(a) Evaluate $\lim_{x \to 0} \dfrac{1-\sqrt{1-x}}{x}$ (b) Find $\dfrac{d}{dx} ln(x\sin(2x))$ (c) Find the derivative of 3^{2x}.

(d) Express cos(sin^{-1} x) in terms of x. (e) Find the Cartesian equation of the locus of the point $\left(\dfrac{3+t}{2-t}, \dfrac{1-3t}{2-t}\right)$.

(f) Use the substitution u = log x, find $\int \dfrac{dx}{x \log x}$ (g) Differentiate $tan^{-1} 2x$ and hence find $\int \dfrac{e^{tan^{-1} 2x}}{1+4x^2}\, dx$

Question 2

(a) Solve the equation $\sin(\theta + 30^0) = 2\cos\theta$, for $0^0 \le \theta \le 360$ (b) Show that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1+\sin\theta}{\cos\theta}$

(c) Show that $\dfrac{1-\sin\theta}{1+\sin\theta} = tan^2\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)$; Hence, or otherwise show that $tan 22\dfrac{1}{2}^0 = \sqrt{2} - 1$

(d) If the period of $f(x) = a + \cos bx$ is 4π and the range is [1,3], find the value of a and the value of b, both $\in R$

(e) Solve for x: $log_2 x^2 - log_4 x = 6$ (f) Use the substitution u = tan x , to evaluate $\int_0^{\pi/4} (1 + tan^2 x) sec^2 x\, dx$

(f) The curve for which $\dfrac{dy}{dx} = a(x-p)(x-q)$, where a ,p and q are constants, has turning points at (2,0) and (1,1).

(i) State the values of p and q. (ii) Use these values to determine the value of a.

Question 3

(a) If x is so small that x^3 and the higher powers of x may be neglected, find an expansion of the form $a + bx + cx^2$ for $(3 - 2x - 2x^2)^5$.

(b) Find the constant term in the expansion of $\left(\dfrac{1}{2x^4} + x^4\right)^4$ (c) Prove by induction that $n^2 + n < n^3$, for $n > 1$,

(d) Find the coefficient of x^n in $(1+x)^{2n}$. By expanding $(1+x)^n$ and $(x+1)^n$, find the coefficient of x^n in $(1+x)^n (x+1)^n$. Hence show that

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \ldots\ldots\ldots + \binom{n}{n}^2 = \binom{2n}{n}$$

(e) Water is poured into an inverted cone, in which the radius is always equal to the height, at a constant rate of 4 cm^3/sec. At what rate is the depth of the water increasing when it is 1 cm deep?

(f) Find the sum to infinity of the series $1 + cos^2 x + cos^4 x + \ldots\ldots + cos^{2(n-1)x}$. Hence find the maximum value of x in the interval $0 < x < \dfrac{\pi}{2}$ for which the sum to infinity is ≥2.

Question 4

(a) The roots of the equation $x^2 + px + q = 0$ are α and $k\alpha$. Show that $\dfrac{(k+1)^2}{k} = \dfrac{p^2}{q}$

(b) If the equations $x^2 + mx + n = 0$ and $x^2 + 2mx + 3n = 0$ have a common root, show that $m^2 = 4n$.

(c) The equation $x^2 - 3x + 1 = 0$ has roots α and β. Find the equation whose roots are $\dfrac{\alpha^3}{\beta}$ and $\dfrac{\beta^3}{\alpha}$.

(d) Let $f(x) = \dfrac{px+q}{x+r}$ where x, p, q ,r are real and $x \neq \pm r$. Find the condition for $f(x)$ to be an even function.

Deduce that if $f(x)$ is an even function then $f(x)$ must reduce to the form $f(x) = k$, where k is a constant.